APPROXIMATE EQUATION OF STATE OF CONDENSED SUBSTANCES

G. A. Bogachev

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The functions entering into the equation of state of Mie–Gruneisen solids are constructed approximately. In [1] an approximate equation of state was proposed for solids whose adiabatic curves obey a linear relationship between the velocity of the shock wave D and the mass velocity U; the slope of the shock adiabatic curve was equal to 1.5. In the present work, the dimensionless variables proposed in [2] are used to construct an equation of state which is free from the above-mentioned limitation. The equation of state found is used to make calculations of the shock compression of porous metals, in particular, of copper, with values of the porosity not differing significantly from unity.

1. Approximate Equation of State of Solids

In [1] an analytical expression is given for the generalized Hugoniot shock adiabatic curve for materials with a linear dependence between the velocity of the shock wave and the mass velocity of the substance, of the form

$$D=a+bU, \tag{1.1}$$

where a and b are experimentally measured constant quantities. This linear connection, as is shown in [2], corresponds to dimensionless variables, making it possible to write the shock adiabatic curve in compact form:

$$p_H=x(1-x)^{-2}. \tag{1.2}$$

Here p_H (the dimensionless pressure at the Hugoniot adiabatic curve) and x are determined by the equalities

$$p_H=P_H/P_c,\ P_c=\rho_0a^2/b,\ x=z/z_L,$$

where $z=1-\rho_0/\rho$ is the relative compressibility of the substance; ρ_0 and ρ are the initial and final densities of the solid; $z_L=1/b$ is the limiting value of the relative compressibility [3]. Thus, the variable x denotes the relative fraction of the compressibility out of the maximally possible; P_c is the so-called characteristic pressure, which can be interpreted either as the resistance of the material to shock compression or as the impact viscosity [2].

The connection, described by the relationship (1.2), between the shock compressibility of the material and the pressure for various materials is represented by exactly the same point in the (P_H, x) plane.

From Eq. (1.1) and the laws of conservation at the front of the shock wave an expression is obtained for the dimensionless specific internal energy at the Hugoniot adiabatic curve in the form

$$\varepsilon_H=1/2xp_H, \tag{1.3}$$

where the notation $\varepsilon_H=E_H/(a^2/b^2)$. In formulas (1.2), (1.3), we neglect the initial pressure and the initial internal energy of the substance ahead of the shock wave.

If the relationship (1.1) is observed up to very large pressures, it then follows from (1.2), (1.3) that the compression of the substance tends, as is required, toward the limiting value $x_*=1$.

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In solids, the specific internal energy and the pressure are usually separated into a thermal part and a cold part (ε_{co}, p_{co}), connected with the deformation of the crystalline lattice. The connection between the thermal pressure and the energy is established by an equation, known as the Mie–Gruneisen equation [4]:

$$\frac{p - p_{co}}{\varepsilon - \varepsilon_{co}} = \frac{\gamma}{b - x}. \tag{1.4}$$

Here p_{co} and ε_{co} are represented in dimensionless form as p_H and ε_H; γ is the Gruneisen coefficient, depending only on the specific volume [5]. From the definition of the variables z and x it follows that the denominator in the right-hand part of (1.4) does not revert to zero (with the exception of the case of the limiting compression of sapphire, for which b = 1; in the remaining solids investigated, b is strictly greater than unity [2]). Since p_{co} and ε_{co} are taken at the isotherm of absolute zero, the following connection will exist between them:

$$d\varepsilon_{co} - p_{co}dx = 0. \tag{1.5}$$

Using Eq. (1.4), which must be satisfied at the Hugoniot adiabatic curve, and relationship (1.5) we obtain the linear differential equation

$$\frac{d\varepsilon_{co}}{dx} - \frac{\gamma}{b - x}\varepsilon_{co} = p_H - \frac{\gamma}{b - x}\varepsilon_H. \tag{1.6}$$

The solution of this equation with the initial condition $\varepsilon_x(0) = 0$ has the form

$$\varepsilon_{co}(x) = e^{\int\limits_0^x \frac{\gamma}{b-x}dx} \int\limits_0^x \left(p_H - \frac{\gamma}{b - x}\varepsilon_H\right) e^{-\int\limits_0^x \frac{\gamma}{b-x}dx} dx.$$

The integrals entering into this formula can be calculated with definite assumptions with respect to the form of the function γ(x). The analytical form of ε_{co}(x), with constant γ and the following partial values of the parameters b = 1.5 and γ = 1 or γ = 2, is given in [1].

If relationships (1.1), (1.4) are valid up to very large pressures, we can find the limiting value γ_*. With compression of a substance by very strong shock waves, starting from some pressure the thermal parts of the pressure and the energy considerably exceed p_{co} and ε_{co}. We therefore have

$$\gamma_* = (b - x)\frac{p_H - p_{co}}{\varepsilon_H - \varepsilon_{co}} \approx (b - x_*)\frac{p_H}{\varepsilon_H} = 2(b - x_*)/x_*. \tag{1.7}$$

Since it has been previously found that $x_* = 1$, then from (1.9) we obtain

$$\gamma_* = 2(b - 1). \tag{1.8}$$

This expression coincides with the analogous formula from [1]. We note that, with compression of a substance by shock waves, the functions of p_{co} and ε_{co} must be known as exactly as possible only up to moderate compressions, since with large compressions their contribution to Eq. (1.4) is negligibly small.

Equation (1.6) can be solved approximately by expansion of the functions entering into it in series in terms of the small parameter x. As a result of integration we have

$$\varepsilon_{co} = \frac{x^2}{2} + \frac{2}{3}x^3 + \frac{1}{12}\left(9 - \frac{\gamma_0}{b}\right)x^4 + \ldots \tag{1.9}$$

From this and from (1.5) we find

$$p_{co} = x + 2x^2 + \left(3 - \frac{\gamma_0}{3b}\right)x^3 + \ldots \tag{1.10}$$

For purposes of comparison, we carry out expansions of ε_H and p_H:

$$\varepsilon_H = 1/2(x^2 + 2x^3 + 3x^4 + \ldots), \quad p_H = x + 2x^2 + 3x^3 + \ldots$$

The coefficients in the series obtained have a simpler form than in the corresponding formulas of [1].

As can be seen from (1.9), (1.10), the constant γ_0 (the first term in the expansion of γ with respect to x) enters only into the coefficients with x^4 and x^3, respectively, in the expressions for ε_{co} and p_{co}, i.e., γ starts to play a significant role only for values of x which are sufficiently close to unity. In view of this, the value of γ in the expression for the elastic energy (1.9) is considerable for moderate compressions, while, with large compressions, the elastic component itself is insignificant. Therefore, in expansion (1.9) we can limit ourselves to the first term and write

$$\varepsilon_{co}=1/2x^2. \tag{1.11}$$

As a basis for approximate constructions of the equation of state of substances, we take this expression and an expansion of p_{co} up to the second order of x, i.e.,

$$p_{co}=x+2x^2. \tag{1.12}$$

In this approximation, from (1.6) for γ we obtain the expression

$$\gamma=2(b-x)(3-2x)/(2-x), \tag{1.13}$$

which, with small and moderate compressions, is almost constant, and with large compressions, tends toward the limiting value γ_* (1.8). Thus, we have completed the construction of all the functions entering into the Mie–Gruneisen equation of state of solids (1.4).

The expressions for ε_{co}, p_{co}, and γ from (1.11)-(1.13) differ from the corresponding formulas of [1]. For example, in the notation of [1], expression (1.11) can be represented in the form $\varepsilon_{co}=1/2b^2\delta^2/(1+\delta)$, while, at the same time, in [1] the following expression is obtained: $\varepsilon_{co}=1.2b^2\delta^2(\delta=z/(1-z))$. Comparison of these relationships shows that, with a sufficiently small value of δ, both formulas should give close results. With other values of δ, the difference may be found to be considerable.

2. Shock Adiabatic Curve of a Porous Substance

Let us use the approximate Mie–Gruneisen equation of state obtained to calculate the shock adiabatic curve of a porous substance. An analytical expression of this adiabatic curve is given in [4] under the assumptions that the electronic pressure and energy are small, the Gruneisen coefficient is constant, and the initial energy can be neglected.

When the compression of a porous substance takes place with a rather large pressure (for porous copper above 26 kbar [6]), the details of the collapse of the pores become insignificant for predicting the final state of the porous substance. If, further, we neglect the contribution of the pores to the total pressure and the internal energy of the system, and use the Mie–Gruneisen equation of state (1.4), then the shock adiabatic curve of a porous substance can be written in the form [6]

$$p_H=\frac{(h-1)\,p_{co}(x)-2\,[\varepsilon_{co}(x)-\varepsilon_T(x_0,\,T_0)]\,/\,(b-x)}{h-\dfrac{b}{m\,(b-x)}}, \tag{2.1}$$

where $h=1+2/\gamma(x)$; T is the absolute temperature; $m=V_0/V_{00}$ is the initial porosity of the substance; V_0 and V_{00} are the initial volumes of monolithic and porous samples, respectively. The subscript zero denotes the initial state. As is well known [4], there exists a limiting volume $V_{lt}=V_{00}/h$, up to which a porous substance can be compressed. If V_p is less than V_0, which is true with a small porosity, when $V_{00}/V_0<h$, then the shock adiabatic curves have a normal course; they lie higher the greater the initial specific volume V_{00}. For copper, with the parameters given in Table 1, (1.13) is used to obtain the evaluation $h_{min}\approx 1.445$. Calculations will be made below of the shock adiabatic curves of porous samples of copper with $V_{00}/V_0<1.21$, i.e., for which the inequality $V_{00}/V_0<h$ is clearly satisfied.

TABLE 1

Substance	ρ_0, g/cm³	a, km/sec	b	U, km/sec	Literature
Cu	8,93	3,94	1,489	0<U<2,5	[6]

TABLE 2

m=1			m=0,88			m=0,82		
U, km/sec	D, km/sec	x	U, km/sec	D, km/sec	x	U, km/sec	D, km/sec	x
0,213	4,257	0,07	0,321	1,945	0,04	0,441	1,875	0,04
0,463	4,629	0,15	0,513	2,702	0,07	0,701	2,695	0,07
0,761	5,073	0,22	0,708	3,291	0,11	0,967	3,392	0,11
1,122	5,611	0,30	0,917	3,822	0,15	1,263	4,073	0,15
1,569	6,276	0,37	1,150	4,341	0,19	1,608	4,799	0,19
2,136	7,121	0,45	1,415	4,881	0,22	2,027	5,631	0,22
2,880	8.228	0,52	1,723	5,469	0,26	2,563	6,658	0,26
			2,086	6,137	0,30			
			2,529	6,929	0,34			
			3,086	7,912	0,37			

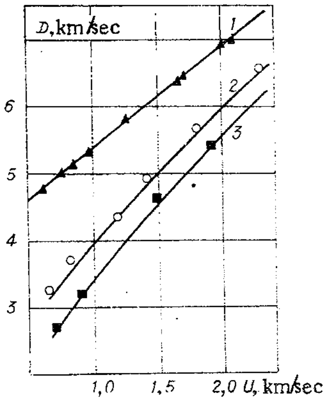

Fig. 1

Let us consider porous samples with a small initial porosity, neglecting the initial thermal energy ε_T in (2.1). Substituting into (2.1) the expressions for p_{c0}, ε_{c0}, and γ from (1.12), (1.13), we represent p_H as a function of x. Using (2.1), we find, for example, shock adiabatic curves for porous copper, with different values of the initial porosity m [6]. The parameters of copper given in Table 1 were used in calculation of these adiabatic curves.

The results of calculations of the Hugoniot shock adiabatic curves, together with the corresponding values of the parameter x, are given in Table 2 in terms of the variables D and U, the transition to which is effected using the laws of conservation at the front of the shock wave: $p_H = DU/V_0$, $z = U/D$. Figure 1 illustrates the dependence of D on U with different values of the initial porosity. Curves 1-3 correspond to the values $m = 1$, 0.88, 0.82. For purposes of comparison, the same figure gives experimental data taken from [5]. As can be seen from the figure, the agreement between the theoretical and experimental numbers is completely satisfactory.

Calculations of shock adiabatic curves, carried out with large values of the porosity, led to results differing from the experimental. Taking account of the thermal energy ε_T in formula (2.1), as is shown in [6], eliminates this divergence.

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